of the point y on γ). γ is then called a periodic orbit of X, of period τ; it is a sub-manifold of M diffeomorphic to the circle S^1.

Finally, in the third case, y is a singular point of X.

1.19. PROPOSITION. Let X be a vector field on the paracompact manifold M. There exists a strictly positive function f on M, of the same differentiability class as X, such that the vector field Y = fX is complete.

Proof. Paracompactness of M implies the existence of a proper function g of class C^s on M. Let $f = \exp(-(Xg)^2)$. If $Y = fX$, then $|Yg| = |(Xg)\exp(-(Xg)^2)| \leq 1$ on M. If c denotes an integral curve of Y defined on a bounded interval J, then $\frac{d}{dt}(g \circ c) = (Yg) \circ c$; hence $|\frac{d}{dt}(g \circ c)| \leq 1$ on J.

Thus the image of $g \circ c$ is bounded and hence the image of c is relatively compact. The proof is concluded by an application of lemma 1.13.

Q.E.D.

1.20. Remark. If c is the maximal integral curve of the vector field X passing through z, and if f is a never vanishing function on M, then the maximal integral curve of Y = fX through z is the map $t \mapsto c(h(t))$, where h is the maximal solution of the differential equation $\frac{ds}{dt} = f(c(s))$ satisfying $h(0) = 0$. Thus these maximal integral curves differ only by a change of parameter, which preserves the orientation for positive f. Hence the orbits of X and Y coincide. We may thus assume, in what follows, the field to be complete (as far as properties of the orbits of a vector field on a paracompact manifold which are invariant with respect to parameter transformations are concerned).

As an example we have